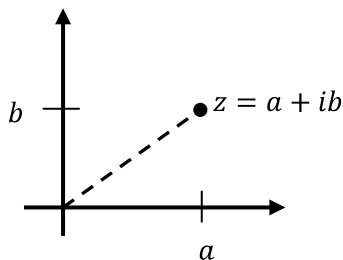
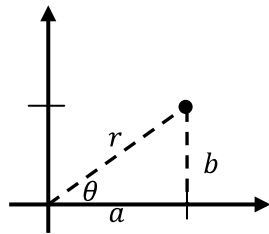


COMPLEX NUMBERS IN POLAR FORM

Rectangular form: $a + ib$



Polar form: $r \operatorname{cis} \theta$



$r = |z|$ = distance of point from origin

θ = argument of z , the angle the line makes with the real axis

From Trigonometry:

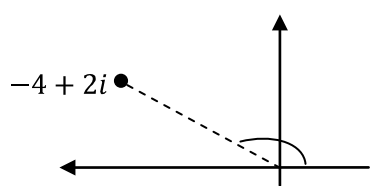
- $\frac{a}{r} = \cos \theta \Rightarrow a = r \cos \theta$
- $\frac{b}{r} = \sin \theta \Rightarrow b = r \sin \theta$
- $\tan \theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$

To convert to polar form:

$$\begin{aligned} a + ib &= r \cos \theta + i r \sin \theta \\ &= r(\cos \theta + i \sin \theta) \\ &= r \operatorname{cis} \theta \end{aligned}$$

Example 1: Write $-4 + 2i$ in polar form.

Sketch $-4 + 2i$ on Argand diagram to check the angle it makes with the real axis:



$$r = |z| = \sqrt{(-4)^2 + 2^2} = \sqrt{20} = 4.47$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{2}{-4}\right) = -26.6^\circ$$

$$\begin{aligned} &= -26.6^\circ + 180^\circ \text{ (2nd quadrant)} \\ &= 153.4^\circ \end{aligned}$$

$$\text{So } -4 + 2i = r \operatorname{cis} \theta = 4.47 \operatorname{cis} (153.4^\circ)$$

On GRAPHICS CALCULATOR:

- To find r : In RUN mode, press OPTN, press F3 for CPLX, press F2 for Abs, then type in the complex number. Your screen should read "Abs $(-4 + 2i)$ ". Press EXE.
- To find θ : In RUN mode, press OPTN, press F3 for CPLX, press F3 for Arg, then type in the complex number. Your screen should read "Arg $(-4 + 2i)$ ". Press EXE.

Example 2: Write $3 \operatorname{cis}(-150^\circ)$ in rectangular form.

$$\begin{aligned} 3 \operatorname{cis}(-150^\circ) &= 3 (\cos(-150^\circ) + i \sin(-150^\circ)) \\ &= 3 \cos(-150^\circ) + i 3 \sin(-150^\circ) \\ &= -2.6 + i(-1.5) \\ &= -2.6 - 1.5i \end{aligned}$$